Exam. Code : 103201
Subject Code : 1029

B.A./B.Sc. Ist Semester

MATHEMATICS

Paper—II (Calculus & Trignometry)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each Section.

SECTION—A

1. (a) Between any two distinct real numbers, there is always an irrational number and therefore, infinitely many irrational numbers. Prove or disprove.

   (b) Prove that

   $$|x + 1| < 2 \text{ iff } \frac{2x-1}{3x+2} \in (-\infty, -\frac{1}{5}) \cup (1, \infty) - \left\{-\frac{2}{3}\right\}.$$

   5,5

2. (a) Prove that $\mathrm{Lt}_{x \to a} \frac{1}{x-a}$ does not exist.

   (b) Let $f(x) = \begin{cases} 1 ; & x \le 3 \\ ax + b ; & 3 < x < 5 \\ 7 ; & 5 \le x \end{cases}$

   Determine the constants a and b so that f may be continuous for all x. 5,5

3. (a) Differentiate $\tan^{-1}(\text{sech } x^2)$ w.r.t. x^2.

(b) If $y = e^{m \sin^{-1} x}$, then $(1 + x^2)y_2 - xy_1 = m^2y$.

(c) If $y = \left(x + \sqrt{1 + x^2}\right)^m$, find, $y_n(0)$. 3,2,5

4. (a) State and prove Taylor's Theorem (with Cauchy's Form of Remainder).

(b) If $f(x) = (1-x)^{\frac{5}{2}}$ and $f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(\theta X)$; $0 < \theta < 1$, find the value of θ as x tends to 1. 5,5

5. (a) Evaluate $Lt_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$.

(b) Show that $f(x) = \frac{1}{x^2}$ is continuous on (0, 1], but it is not uniformly continuous on (0, 1]. Is 'f' uniformly continuous on [a, 1], if a > 0 ? 5,5

SECTION—B

6. (a) If α, β be roots of $t^2 - 2t + 2 = 0$ then prove that

$$\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \frac{\sin n\varphi}{\sin^n \varphi}.$$

(b) Prove that n, nth roots of unity form a G.P. Also show that their sum is zero and product is equal to $(-1)^{n-1}$. 5,5

7. (a) Prove that :

$$[\sin(\alpha-\theta)+e^{\pm i\alpha}\sin\theta]^n=\sin^{n-1}\alpha[\sin(\alpha-n\theta)+e^{\pm i\alpha}\sin n\theta]$$

(b) Prove that :

$$i^{i^i}=\cos\theta+i\sin\theta,\ \text{where}\ \theta=(4m+1)\frac{\pi}{2}e^{-(4n+1)\frac{\pi}{2}};m,n\in Z$$

5,5

8. (a) If $\tan\frac{x}{2}=\tanh\frac{x}{2}$, prove that cos x cos h x = 1.

(b) If sin (u + iv) = x + iy, prove that :

(i) $\frac{x^2}{\cosh^2 v}+\frac{y^2}{\sinh^2 v}=1$

(ii) $\frac{x^2}{\sin^2 u}-\frac{y^2}{\cos^2 u}=1$ 5,5

9. (a) Separate into real and imaginary parts :

$\tan^{-1}(x+iy)$

(b) Sum to n terms the series :

$\cos\alpha+\cos 2\alpha+\cos 3\alpha+\cdots$ n terms.

Deduce the sum $1^2+2^2+3^2+\ldots+n^2$. 5,5

10. (a) Sum to infinity the series :

$\tan\alpha\ \tan(\alpha+\beta)+\tan(\alpha+\beta)\ \tan(\alpha+2\beta)+\tan(\alpha+2\beta)\ \tan(\alpha+3\beta)+\ldots\ \infty$

(b) Prove that $Lt_{x\to 0}\frac{1}{x^2}\log\left(\frac{\tan^{-1}x}{x}\right)=-\frac{1}{3}$. 5,5